

Tablix timetabling model

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Abstract

This document gives a formal description in mathematical language of the type of timetabling problems that is supported by the Tablix kernel.

1 Definitions

1.1 Resource type, timetable, fitness value

Let \mathcal{T}_0 through \mathcal{T}_{M-1} be sets of *resources* $R_{i,j}$ called *resource types*.

$$\begin{aligned}\mathcal{T}_0 &= \{R_{0,0}, R_{0,1}, R_{0,2} \dots R_{0,n_0-1}\} \\ \mathcal{T}_1 &= \{R_{1,0}, R_{1,1}, R_{1,2} \dots R_{1,n_1-1}\} \\ \mathcal{T}_2 &= \{R_{2,0}, R_{2,1}, R_{2,2} \dots R_{2,n_2-1}\} \\ &\vdots \\ \mathcal{T}_{M-1} &= \{R_{M-1,0}, R_{M-1,1}, R_{M-1,2} \dots R_{M-1,n_{M-1}-1}\}\end{aligned}$$

We can then define a *timetable* B as a set of N tuples of M resources.

$$B = \left\{ \langle r_{i,0}, r_{i,1} \dots r_{i,M-1} \rangle; \begin{array}{l} i = 0, 1, 2 \dots N-1 \\ r_{i,0} \in \mathcal{T}_0, r_{i,1} \in \mathcal{T}_1 \dots r_{i,M-1} \in \mathcal{T}_{M-1} \end{array} \right\} \quad (1)$$

A tuple in the timetable is composed of M resources, one from each resource type and there are N tuples in the timetable.

Timetable search space B^* is defined as a set of N tuples of M *domains*.

$$B^* = \left\{ \langle \mathcal{D}_{i,0}, \mathcal{D}_{i,1} \dots \mathcal{D}_{i,M-1} \rangle; \begin{array}{l} i = 0, 1, 2 \dots N-1 \\ \mathcal{D}_{i,0} \subseteq \mathcal{T}_0 \dots \mathcal{D}_{i,M-1} \subseteq \mathcal{T}_{M-1} \end{array} \right\} \quad (2)$$

A timetable search space tells the problem solving algorithm which resources within each resource type can be included in a tuple. Each tuple can include at least one resource of each resource type.

A resource type \mathcal{T}_j is said to be a *constant resource type* for the timetable search space B^* if

$$\text{card}\mathcal{D}_{i,j} = 1 \quad \forall i \in [0, N-1] \quad (3)$$

and a *variable resource type* otherwise.

A timetable B is said to be an element of the timetable search space B^* if the following is true:

$$B \in B^* \iff r_{i,j} \in \mathcal{D}_{i,j} \quad \forall i \in [0, N-1], \forall j \in [0, M-1] \quad (4)$$

Partial fitness functions f'_0 through $f'_{K'-1}$ and f''_0 through $f''_{K''-1}$ are defined as functions that associate a timetable B with non-negative integers called *partial fitness values*.

$$\begin{aligned} f'_i &= f'_i(B) \\ f''_j &= f''_j(B) \end{aligned}$$

A partial fitness function can be either *mandatory* f'_i or *non-mandatory* f''_j .

A *fitness function* of a timetable B is defined as the weighted sum of all defined partial fitness functions:

$$f(B) = \sum_{i=0}^{K'-1} W'_i \cdot f'_i(B) + \sum_{j=0}^{K''-1} W''_j \cdot f''_j(B) \quad (5)$$

Timetabling algorithm is searching for a global minimum of the fitness function. The ratio between weight values for different partial fitness functions defines how much computational effort goes into reducing a value of a certain partial fitness function. The set of partial fitness functions depends on which modules are used (typically each module defines one partial fitness function).

A *solution function* of a timetable B is defined as:

$$s(B) = \begin{cases} 0 & \sum_{i=0}^{K'-1} f'_i(B) > 0 \\ 1 & \sum_{i=0}^{K'-1} f'_i(B) = 0 \end{cases} \quad (6)$$

Solution function tells the algorithm when an acceptable solution to the timetabling problem is found.

1.2 Timetabling problem

A *timetabling problem* is defined as:

$$TP = \langle \mathcal{T}, B^*, f, s \rangle \quad (7)$$

Where \mathcal{T} is a set of resource types, B^* is the timetable search space, f is the fitness function and s is the solution function.

A *solution to the timetabling problem* TP is a timetable B such that the following is true:

$$B \in B^* \quad (8)$$

$$s(B) = 1 \quad (9)$$

An *optimal solution* to the timetabling problem TP is a timetable B_o that also satisfies the following condition:

$$f(B_o) \leq f(B) \quad \forall B \in B^* \quad (10)$$