0.1 Resource type, timetable, fitness value

Let $\mathcal{T}_0$ through $\mathcal{T}_{M-1}$ be sets of *resources* $R_{i,j}$ called *resource types.*

$$\begin{aligned}
\mathcal{T}_0 &= \{R_{0,0}, R_{0,1}, R_{0,2} \dots R_{0,n_0-1}\} \\
\mathcal{T}_1 &= \{R_{1,0}, R_{1,1}, R_{1,2} \dots R_{1,n_1-1}\} \\
\mathcal{T}_2 &= \{R_{2,0}, R_{2,1}, R_{2,2} \dots R_{2,n_2-1}\} \\
&\vdots \\
\mathcal{T}_{M-1} &= \{R_{M-1,0}, R_{M-1,1}, R_{M-1,2} \dots R_{M-1,n_M-1}\}
\end{aligned}$$

We can then define a *timetable* B as a set of N tuples of M resources.

$$B = \left\{ \langle R_{i,0}, R_{i,1} \dots R_{i,M-1} \rangle \, ; \begin{array}{l} i \in [0, N-1] \\ R_{i,0} \in \mathcal{T}_0, R_{i,1} \in \mathcal{T}_1 \dots R_{i,M-1} \in \mathcal{T}_{M-1} \end{array} \right\} \tag{1}$$

Timetable search space B^* is defined as a set of N tuples of M *domains.*

$$B^* = \left\{ \langle \mathcal{D}_{i,0}, \mathcal{D}_{i,1} \dots \mathcal{D}_{i,M-1} \rangle \, ; \begin{array}{l} i \in [0, N-1] \\ \mathcal{D}_{i,0} \subseteq \mathcal{T}_0 \dots \mathcal{D}_{i,M-1} \subseteq \mathcal{T}_{M-1} \end{array} \right\} \tag{2}$$

A resource type $\mathcal{T}_j$ is said to be a *constant resource type* for the timetable search space B^* if

$$\mathrm{card}\mathcal{D}_{i,j} = 1 \qquad \forall i \in [0, N-1] \tag{3}$$

and a *variable resource type* otherwise.

A timetable B is said to be an element of the timetable search space B^* if the following is true:

$$B \in B^* \Longleftrightarrow R_{i,j} \in \mathcal{D}_{i,j} \quad \forall i \in [0, N-1], \forall j \in [0, M-1] \tag{4}$$

Partial fitness functions f'_0 through $f'_{K'-1}$ and f''_0 through $f''_{K''-1}$ are defined as functions that associate a timetable B with non-negative integers called *patial fitness values.*

$$\begin{aligned}
f'_i &= f'_i(B) \\
f''_j &= f''_j(B)
\end{aligned}$$

A partial fitness function can be either *mandatory* f'_i or *non-mandatory* f''_j.

A *fitness funtion* of a timetable B is defined as the weighted sum of all defined partial fitness functions:

$$f(B) = \sum_{i=0}^{K'-1} W'_i \cdot f'_i(B) + \sum_{j=0}^{K''-1} W''_j \cdot f''_j(B) \tag{5}$$

A *solution function* of a timetable B is defined as:

$$s(B) = \left\{ \begin{array}{ll} 0 & \sum_{i=0}^{K'-1} f'_i(B) > 0 \\ 1 & \sum_{i=0}^{K'-1} f'_i(B) = 0 \end{array} \right. \tag{6}$$

0.2 Timetabling problem

A *timetabling problem* is defined as:

$$TP = \langle \mathcal{T}, B^*, f, s \rangle \tag{7}$$

Where $\mathcal{T}$ is a set of resource types, B^* is the timetable search space, f is the fitness function and s is the solution function.

A *solution to the timetabling problem* TP is a timetable B such that the following is true:

$$B \in B^* \tag{8}$$
$$s(B) = 1 \tag{9}$$

An *optimal solution* to the timetabling problem TP is a timetable B_o that also satisfies the following condition:

$$f(B_o) \leq f(B) \quad \forall B \in B^* \tag{10}$$