Simulating effects of non-Gaussian noise on covariance-based detectors

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***Abstract*—In radio receiver design, only total additive noise power is typically considered. Non-Gaussian noise components, such as spurious signal crosstalk, are considered insignificant if their power is below thermal noise. However in spectrum sensing tasks, non-Gaussian components can significantly impair detector performance, even at otherwise insignificant powers. In this paper we present a numerical simulation to estimate the impact of such impairments on minimal detectable signal power. We use the simulation to estimate the effect of a weak spurious cosine signal and digital down-conversion on the performance of two sample covariance-based detectors for use in an embedded sensor.**

I. Introduction

All analog circuits exhibit a certain amount of noise and radio frequency front-ends are no exception. Johnson-Nyquist thermal noise is unavoidable even in an ideal case due to laws of physics. Practical devices typically have additional noise sources at various points in the circuit that all contribute to the total noise power added to the received signal.

In a typical use case, a radio front-end is coupled with demodulator to form a receiver that recovers a symbol stream from a radio frequency signal. In vast majority of cases, additive noise is orthogonal to the demodulation process. It is very unlikely for noise sources to be coherent with the received signal. Therefore it is typically enough for noise to only be described by its mean power. For example, a signal-to-noise power ratio (SNR) is a good predictor for the symbol-error rate. This is also the reason why noise performance of different radio front-ends is often compared using metrics based on additive noise power, like noise figure and noise temperature.

Spectrum sensing, however, differs significantly from demodulation in this respect. Advanced signal detection methods estimate stochastic properties of the signal using statistical approximations. There is no coherent detection. Methods work on the principle that statistically, an information-carrying signal differs from noise. One such statistical property that is commonly exploited for this purpose is sample covariance, for instance in covariance-based[1], eigenvalue-based[2] and cyclostationary detectors[3]. These methods can theoretically detect the presence of weak, unknown, man-made signals even when their SNR is well below 1. However, their operation is based on the assumption that noise samples are i.i.d. Gaussian (i.e. only frequency independent white noise is added to the signal). In the presence of non-Gaussian noise, their performance normally degrades.

Thermal noise is Gaussian for all practical purposes. Other noise sources in a practical receiver, however, are not. Nonlinear components have sources of noise that are fundamentally frequency-dependent. Even frequency-independent Gaussian noise that is injected near the input of the circuit typically gets shaped by subsequent filtering stages into a noise component with frequency-dependent spectral power density and highly correlated values. Another such example is crosstalk between different parts of the circuit, for instance from the local oscillator PLL or digital signal lines, which might contribute periodic components to the noise. The latter is especially common in modern designs where analog and digital circuits are present on the same circuit board or increasingly even on the same silicon die.

It follows from the above that design targets for a radio front-end that is to perform well with spectrum sensing are quite different from a device optimized for signal reception. In the former case, phase properties of the additive noise must also be taken into account and noise figure alone is no longer a good figure of merit. For instance, it might be more optimal for a spectrum sensing receiver to trade in higher overall noise power for noise that has more favorable statistical properties.

In this work we aim to evaluate the effect of non-Gaussian noise sources on the performance a covariance-based detector in the context of a channel occupancy decision. More specifically, we limit ourselves to covariance absolute value (CAV) and maximum auto-correlation (MAC) detectors[4], which have been previously found to have good performance in simulations[5] and practical use cases. As a metric of performance we choose minimal detectable signal power P_{in-min} at the chosen minimal probability of detection $P_{d-min} = 0.9$ and probability of false alarm $P_{fa} = 0.1$. As a test signal we use the IEEE wireless microphone signal test vector[6], which is a common test vector used to compare the performance of spectrum sensing methods.

Using a numerical simulation, described in Section II, we estimate the effect of two common sources of non-Gaussian noise we encountered in laboratory experiments. We verify the simulation by comparing results of control simulation campaigns against analytical calculations in Section III. Results of the simulation are presented in Section IV. Finally, we draw conclusions from results in Section V and give some design guidelines for spectrum sensing receivers.

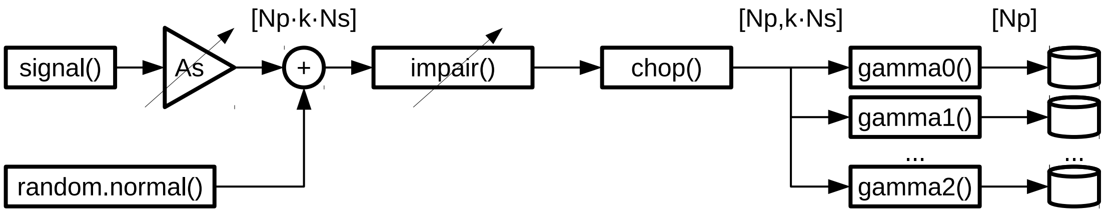

Fig. 1. Block diagram of the basic simulation process `process(As, impair)`.

II. Simulation method

We simulated a detector with the following parameters. Values used correspond to the high-end embedded sensor profile we evaluated in [7].

- Size of signal sample buffer $N_s = 25000\mathrm{samples}$,
- sampling rate $f_s = 2\mathrm{Msample/s}$.

To obtain a statistically valid estimate of probability of detection, we performed $N_p = 1000$ repetitions of simulation for each set of input variable values.

To implement our simulation we chose NumPy and SciPy toolkits[8] for numerical processing in Python. Code used for simulations is available on-line[1].

The basic simulation process is illustrated in Fig.1. The process generates an ideal signal vector of length $N_p \cdot N_s$ using the `signal()` function. This vector represents time-domain samples of the signal being detected, sampled with f_s. In our case, `signal()` implemented the IEEE wireless microphone signal test vector[6]. Generated signal has mean power 1 and central frequency $f_{mic} = f_s/4$. Signal samples are then multiplied by a constant `As`$= \sqrt{P_{in}}$ to set the signal power P_{in}.

This ideal signal is then summed with a vector of Gaussian noise samples obtained using the `numpy.random.normal()` function. This function uses a Mersene Twister pseudo-random number generator and the Box-Muller transform to obtain a $N_p \cdot N_s$ vector of normally-distributed random values. This step represents the addition of thermal noise. The noise level is set to constant $-100\mathrm{dB}$ (i.e. $\sigma_w = 10^{-5}$).

The signal is then passed to the `impair()` function, which adds non-Gaussian noise components to our signal vector.

The signal vector of length $N_p \cdot N_s$ is finally reshaped by the `chop()` function into N_p vectors of length N_s. These vectors are then passed into `gamma()` functions. These functions calculate the test statistic γ according to different spectrum sensing methods. Each function takes all N_p signal vectors and produces N_p values. These values are then stored into files for further processing.

Thus a single run of our process simulates N_p repetitions of signal detection with each of the `gamma()` functions for one value of `As` and one `impair()` function. To efficiently run the simulation on a symmetric-multiprocessing machine, up to 4 such processes were run simultaneously using a process pool implemented in the `multiprocessing` module.

[1] https://github.com/avian2/spectrum-sensing-methods

In this work we used the following `impair()` functions:

- Add a cosine signal. Functions that add signal at frequencies $f_n = 750$, 501, 500, 250, 62.5 and $15.625\mathrm{kHz}$ and powers P_n between -130 and $-100\mathrm{dB}$ were used.
- Simulate ADC oversampling with digital down-conversion. `scipy.signal.decimate()` function was used for decimation, which implements an order 8 Chebyshev type I filter. Decimation factors k between 1 and 8 were used. In oversampling simulations, N_s and f_s were adjusted accordingly. Since down-conversion also decreases total noise power due to decreased bandwidth, σ_w was adjusted so that total noise power after decimation remained constant $-100\mathrm{dB}$.
- Null function. Used for validation.

For each `impair()` function being tested, the simulation process was ran with values of `As` between from -130 to $-110\mathrm{dB}$ in $1\mathrm{dB}$ steps as well as with `As` set to 0 (i.e. calculating γ with only noise component present).

To estimate the minimal signal power P_{in-min} for each pair of `impair()` and `gamma()` functions, first the calculated values of γ for `As`$= 0$ were processed. They were used to estimate the complementary cumulative distribution function (CCDF) for the noise-only case. The CCDF was used to obtain the threshold γ_0, given the desired P_{fa}[7].

Given γ_0, we estimated the probability of detection P_d for each simulation run where `As`> 0. $P_d \approx \frac{N_1}{N_p}$ where N_1 is the number of γ values above γ_0.

To estimate P_{in-min} we then used (`As`, P_d) pairs obtained and used interpolation to calculate the `As` = P_{in-min} value where $P_d = P_{d-min}$.

III. Validation

First step in validating our results was to check if sample covariance of pure Gaussian noise fits with theoretical prediction. Noise samples should be i.i.d. and therefore have zero covariance. Fig.2 shows estimated covariance values for time lag l from 0 to 24 using $N_p \cdot N_s$ samples. Variance ($l = 0$) is 10^{-10} and fits our chosen Gaussian noise power σ_w^2. It can also be seen that covariances with $l > 0$ have amplitudes approximately 4 orders of magnitude below σ_w^2. Due to the finite length of the signal vector we cannot expect the pseudo-random generator to yield exactly 0 covariance. We consider this close enough to the theoretical prediction for the purpose of our simulation.

For validation purposes, we ran an extra simulation campaign for each detector without any signal impairments (i.e.

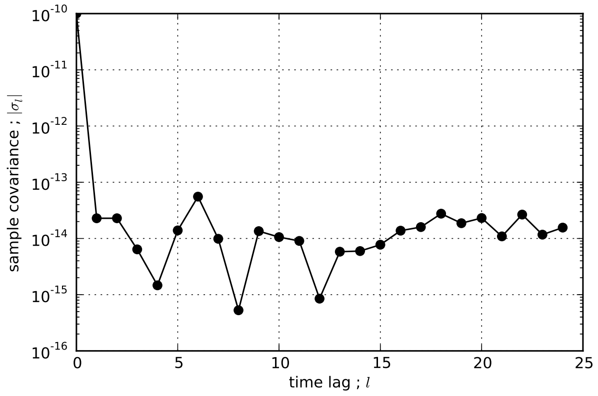

Fig. 2. Simulated Gaussian noise sample covariance.

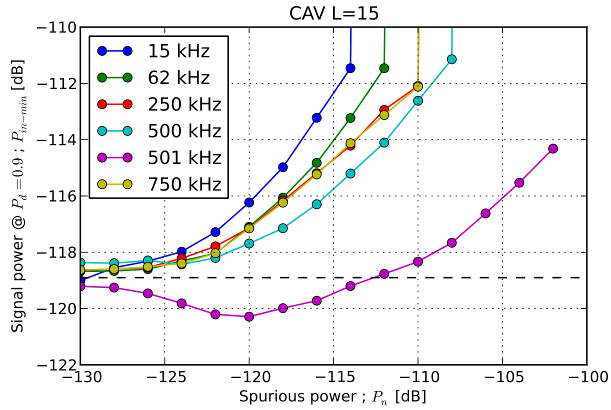

Fig. 4. Effect of spurious cosine noise component on the CAV detector. Dashed black line shows P_{in-min} without impairments.

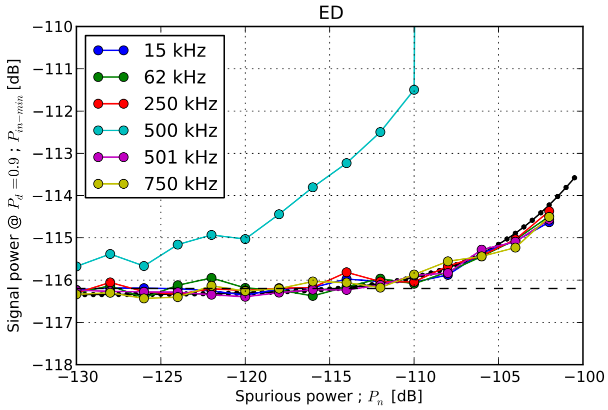

Fig. 3. Effect of spurious cosine noise component on energy detection. Small black dots shows the analytical result. Dashed black line shows P_{in-min} without impairments.

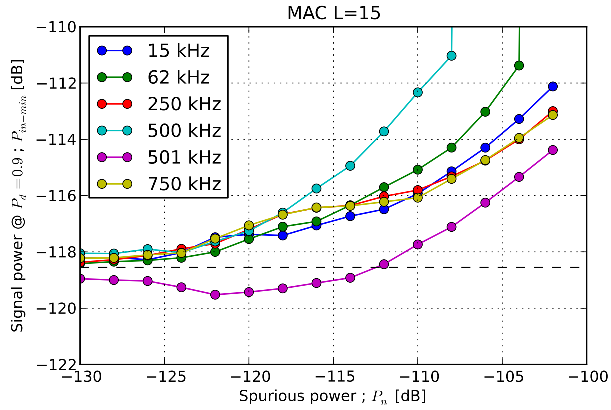

Fig. 5. Effect of spurious cosine noise component on the MAC detector. Dashed black line shows P_{in-min} without impairments.

setting `impair()` to null). If the simulation was operating correctly we expect results of campaigns with impairments to converge to the non-impaired result when the effect of the impairment is decreased (e.g. spurious power P_n or decimation factor k).

In addition to simulating the effect of impairments on covariance-based detectors we also ran all simulations for the energy detector. Energy detector is well known and allows for analytical calculation of P_{in-min}. Results of the spurious cosine noise component for energy detection is shown in Fig.3.

It can be seen from the figure that when P_n decreases, P_{in-min} values converge to the result without the impairment, which fits our prediction. The result also fits well with the analytical calculation of P_{in-min}. The only campaign that does not fit is when $f_n = 500\text{kHz}$. This is due to the fact that the signal test vector also contains a signal at that frequency ($f_n = f_s/4 = f_{mic}$). Hence the spurious and the test vector are no longer orthogonal, which is one of the assumptions of the theoretical prediction.

IV. Results

Results of the simulation of the spurious cosine signal for the CAV and MAC detectors are shown in Fig.4 and 5 respectively. Smoothing factor for the detectors shown was $L = 15$.

Again we can see that P_{in-min} converges to the non-impaired result as expected when P_n decreases. It is also evident that both covariance-based detectors are more sensitive to the spurious signal than energy detection. While energy detector tolerated up to -112dB of spurious power without significant degradation, covariance-based detectors were deviating from the non-impaired case already at -126dB. Even at -114dB, performance of the CAV detector for all spurious frequencies was worse than that of energy detection. Performance of the MAC detector degraded slower than that of CAV, surpassing performance of energy detection at approximately -110dB.

Frequency of the spurious signal affects the degradation as well. From these simulations it is not clear whether higher or lower frequencies are worse. MAC detector seems to perform

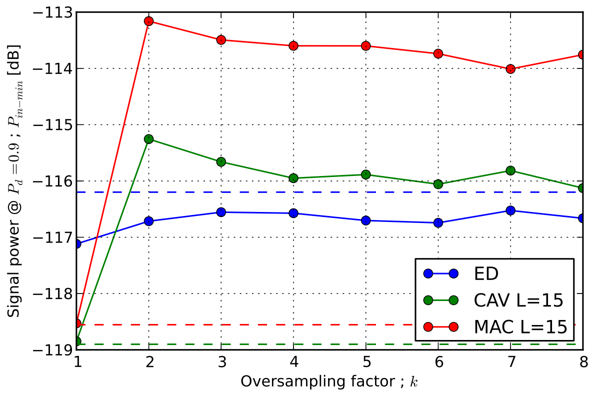

Fig. 6. Effect of oversampling on energy, CAV and MAC detectors. Dashed lines show P_{in-min} without impairments.

more consistently across the tested frequencies, although more data points would be needed for a conclusive results. Considering that the frequency of the spurious signal is related to the maximum time lag at which the spurious affects are significant, it is possible that there is a relation between smoothing factor and the tolerance of the detector to certain frequencies. However this relation was not evident from our simulations.

Similar to what we have seen in energy detection, when the frequency of the spurious signal is at or near one of the frequencies present in the test vectors, the results are anomalous. In both CAV and MAC detectors, a weak spurious component at $f_n = 501\text{kHz} = f_{mic} + 1\text{kHz}$ and below $P_n = -112\text{dB}$ actually improved the performance of the detector. MAC detector had worse performance with $f_n = 500\text{kHz} = f_{mic}$ than for other spurious frequencies. The exact reason for this effect is not known, but likely due to interference effects between the spurious signal and components of the test vector.

Results of the digital down-conversion simulation are shown in Fig.6 for energy detection, CAV and MAC detectors. Results are again shown for smoothing factor $L = 15$.

It can be seen that minimal detectable signal power for CAV and MAC detectors increases approximately -3dB and -5dB respectively if the input signal is oversampled. The value of the oversampling factor k does not significantly affect the result if. As expected, when $k = 1$, the simulation gives the same result as in the unimpaired case.

As expected, the result of the oversampling simulation for the energy detection shows no relation between P_{in-min} and k. However, results are approximately 0.5dB below the expected value. The reason for this discrepancy is now known. As mentioned in Section II, the noise variance σ_w was adjusted based on k in this simulation. It is possible that numerical errors in this adjustment caused the anomalous energy detection results.

V. Conclusions

We have shown that performance of covariance-based detectors is significantly impaired by non-Gaussian components in additive receiver noise.

Power of constant-wave cosine spurious signals in the receiver should be kept below -30dB compared to the thermal noise level, if performance of covariance-based detection is not to be impaired. At -10dB, when the spurious power level would still be considered insignificant in a typical design, the minimal detectable power can increase for more than 10dB in the worst case. This is true even when the frequency of the spurious signal low compared to the sampling frequency.

If the presence of these spurious signals cannot be prevented, performance of the MAC detector degrades slower than that of CAV detector.

If the frequency of the spurious cosine signal happens to coincide with the received signal, interference effects cause anomalous detection performance. This means that detector with such impairments does not have consistent detection performance over its entire frequency range. It will be either more or less sensitive to signals that contain frequencies near the frequency of the spurious signal.

It is possible that effect of such spurious signals can be reduced by the correct choice of the detector smoothing factor, however more simulations would be needed to confirm this.

Digital down-conversion increases the minimal detectable signal for both CAV and MAC detectors by average 4dB. The effect is largely independent of the oversampling factor k. CAV detector is slightly more resistant to this impairment.

However, it should be noted that this effect is small compared to typical SNR gains due to oversampling in radio front-ends employing DDC. It is likely that a final design employing oversampling will perform better even with this negative effect on detector performance. Furthermore, digital down-conversion introduces covariances into noise samples due to linear filtering. This effect can in theory be compensated for in the detector using a prewhitening technique described in [1]. Further simulations would be necessary, however, to confirm the effectiveness of prewhitening.

References

[1] Y. Zeng and Y. C. Liang, “Spectrum-Sensing Algorithms for Cognitive Radio Based on Statistical Covariances,” in *IEEE Transactions on Vehicular Technology*, vol. 58, pp. 1804–1815, 2009.

[2] Y. Zeng and Y. C. Liang, “Eigenvalue-Based Spectrum Sensing Algorithms for Cognitive Radio,” *IEEE Transactions on Communications*, vol. 57, no. 6, pp. 1784–1793, 2009.

[3] G. B. Giannakis, “Cyclostationary Signal Analysis,” in *Digital Signal Processing Handbook* (V. K. Madisetti and D. B. Williams, eds.), CRC Press LLC, 1999.

[4] Y. Zeng and Y.-C. Liang, “Robust spectrum sensing in cognitive radio,” in *IEEE International Symposium on Personal, Indoor and Mobile Radio Communications Workshops*, pp. 1–8, IEEE, 2010.

[5] R. Dionísio *et al.*, “Sensing algorithms for TVWS operations,” public deliverable, COGEU project (FP7 ICT-2009.1.1), 2011.

[6] C. Clanton, M. Kenkel, and Y. Tang, “Wireless Microphone Signal Simulation Method.” IEEE 802.22-07/0124r0, 2007.

[7] T. Šolc, “Spectrum sensing methods and implementations,” seminar paper, Jožef Stefan International Postgraduate School, 2014.

[8] Numpy developers, “Numpy.” http://www.numpy.org/, 2013.