Effects of non-Gaussian noise on covariance-based detectors

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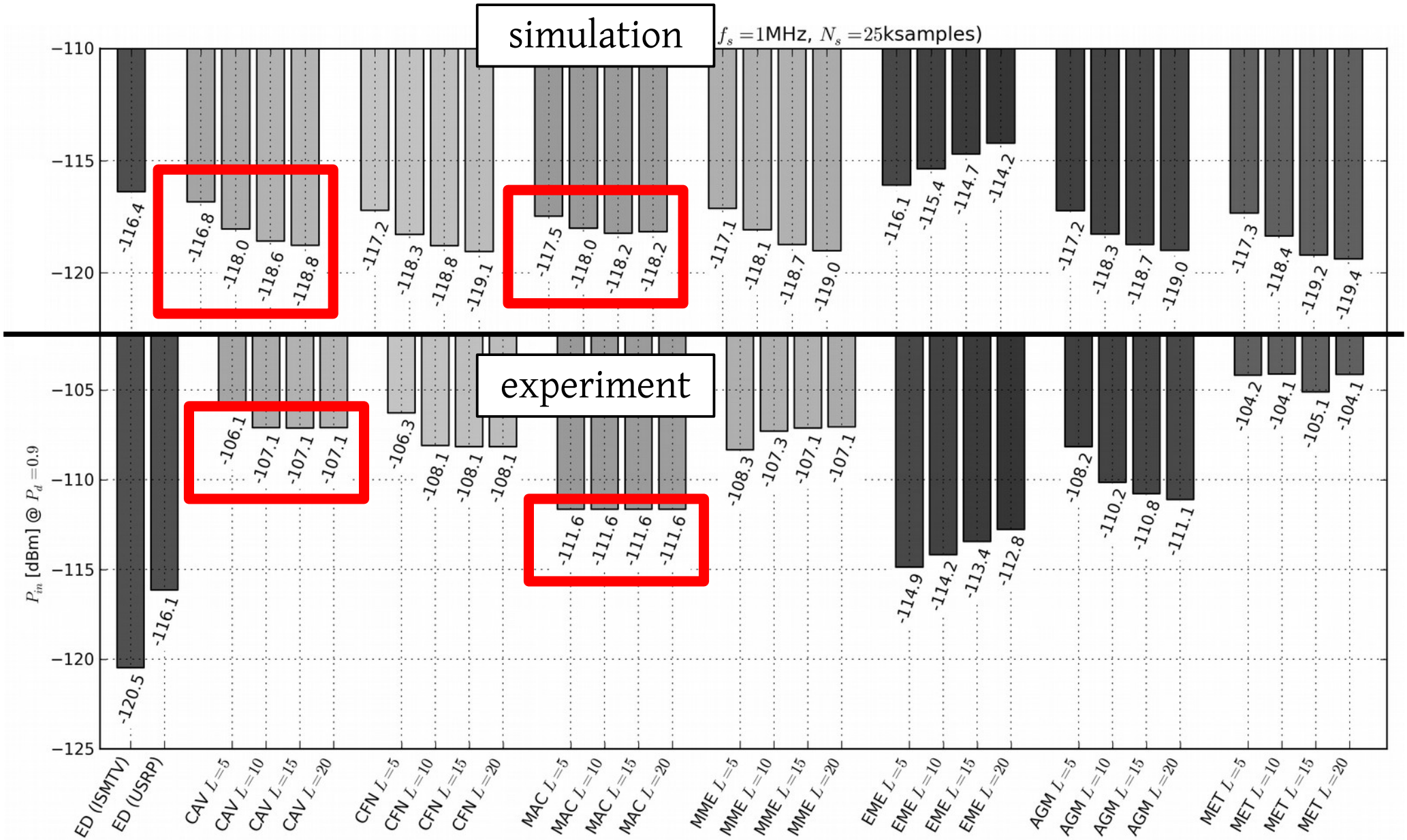
Introduction

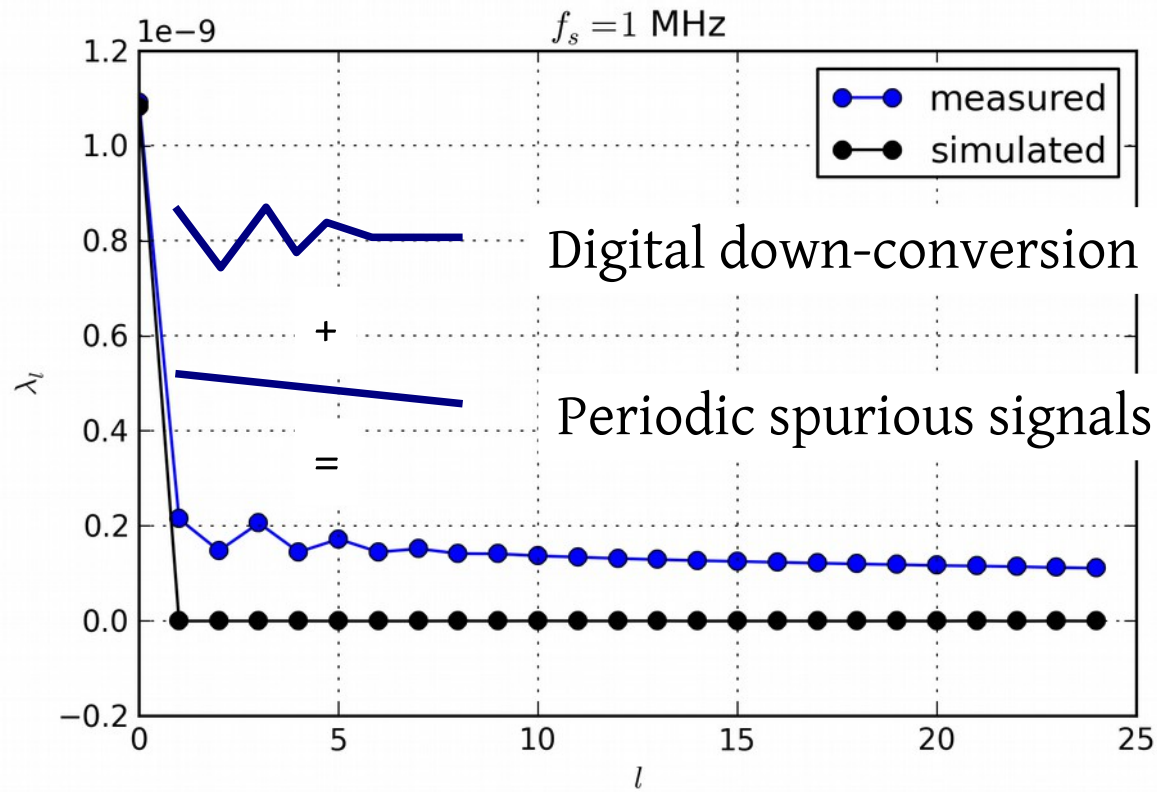
- All radio receivers exhibit additive noise
  - Johnson-Nyquist (thermal) noise, signal crosstalk, etc.
  - Random (Gaussan, non-Gaussian) and deterministic
  - Typically only total noise power is considered in design (i.e. noise figure)
- Spectrum sensing and occupancy detection
  - Several popular methods (CBD, EBD, cyclostationary, ...) exploit sample covariance for detection of weak signals.
  - Based on assumption that noise samples are i.i.d.
  - **Statistical properties of receiver noise become important as well as total added power.**

Motivation


simulation
experiment
f_s =1MHz, N_s =25ksamples)
P_{in} [dBm] @ P_d =0.9
−110
−115
−120
−105
−125
-116.4
-116.8
-118.0
-118.6
-118.8
-117.2
-118.3
-118.8
-119.1
-117.5
-118.0
-118.2
-118.2
-117.1
-118.1
-118.7
-119.0
-116.1
-115.4
-114.7
-114.2
-117.2
-118.3
-118.7
-119.0
-117.3
-118.4
-119.2
-119.4
-120.5
-116.1
-106.1
-107.1
-107.1
-107.1
-106.3
-108.1
-108.1
-108.1
-111.6
-111.6
-111.6
-111.6
-108.3
-107.3
-107.1
-107.1
-114.9
-114.2
-113.4
-112.8
-108.2
-110.2
-110.8
-111.1
-104.2
-104.1
-105.1
-104.1
ED (ISMTV)
ED (USRP)
CAV L=5
CAV L=10
CAV L=15
CAV L=20
CFN L=5
CFN L=10
CFN L=15
CFN L=20
MAC L=5
MAC L=10
MAC L=15
MAC L=20
MME L=5
MME L=10
MME L=15
MME L=20
EME L=5
EME L=10
EME L=15
EME L=20
AGM L=5
AGM L=10
AGM L=15
AGM L=20
MET L=5
MET L=10
MET L=15
MET L=20

Working hypothesis

- Bad CBD performance in experiment compared to simulated ideal case is due to non-Gaussian noise.

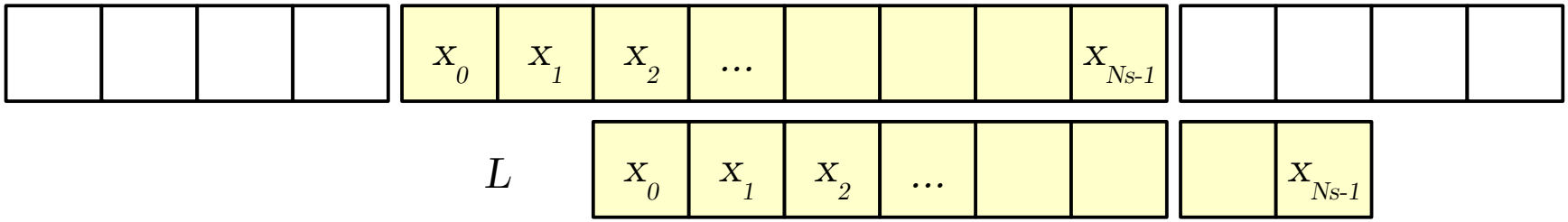


Goals

- Estimate the effect of non-Gaussian receiver noise
  - on CAV, MAC (covariance-based) detectors
  - metric of detector performance: $P_{in\text{-}min}$ @ fixed P_{fa}, P_d
  - detected signal: IEEE wireless microphone signal test vector
- Considered sources of non-Gaussian noise
  - Clock or other constant-wave signal cross-talk,
  - thermal noise, shaped by digital down-conversion.
- Determine basic guidelines for receiver design
  - What is the best compromise between non-Gaussian and Gaussian noise?

Covariance-based detector (CBD)

$$\sigma_l = \frac{1}{N_s} \sum_{n=0}^{N_s-1} x_n \cdot x_{n-l} \qquad l \in [0, L-1]$$



$$\mathbf{R} = [r_{ij}] = \begin{bmatrix} \sigma_0 & \sigma_1 & \dots & \sigma_{L-1} \\ \sigma_1 & \sigma_0 & \dots & \sigma_{L-2} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{L-1} & \sigma_{L-2} & \dots & \sigma_0 \end{bmatrix}$$

ZENG , Y., AND LIANG , Y. C. Spectrum-Sensing Algorithms for Cognitive Radio Based on Statistical Covariances. In *IEEE Transactions on Vehicular Technology* (2009), vol. 58, pp. 1804–1815.

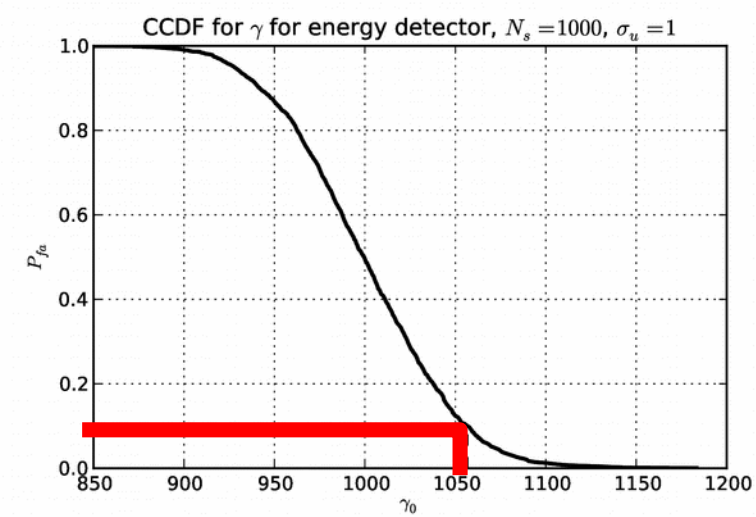
Covariance-based detector (CBD)

CAV $$T_1 = \frac{1}{L}\sum_{i=1}^{L}\sum_{j=1}^{L}|r_{ij}| \qquad T_2 = \frac{1}{L}\sum_{i=1}^{L}|r_{ii}|$$

MAC $$T_1 = \max_{i\neq j}|r_{ij}| \qquad T_2 = \frac{1}{L}\sum_{i=1}^{L}|r_{ii}|$$

$$\boxed{\gamma = \frac{T_1}{T_2}}$$

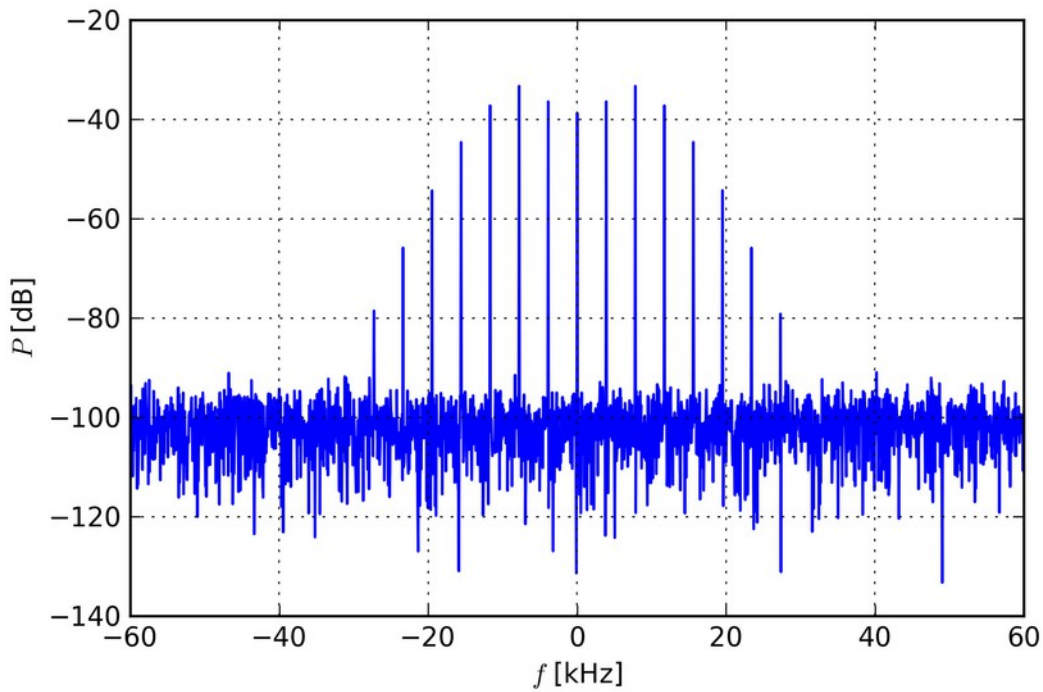
- Calculate a test statistic $\gamma = \gamma(\mathbf{R})$ from covariance matrix.
- Based on P_{fa}, determine γ_0.
- Channel is occupied if $\gamma > \gamma_0$.



ZENG , Y., AND LIANG , Y. C. Robust spectrum sensing in cognitive radio.
In *IEEE International Symposium on Personal, Indoor and Mobile Radio Communications Workshops* (2010), IEEE, pp. 1–8.

Test signal

- tone-modulated FM carrier
  - “soft speaker” IEEE wireless microphone signal test vector

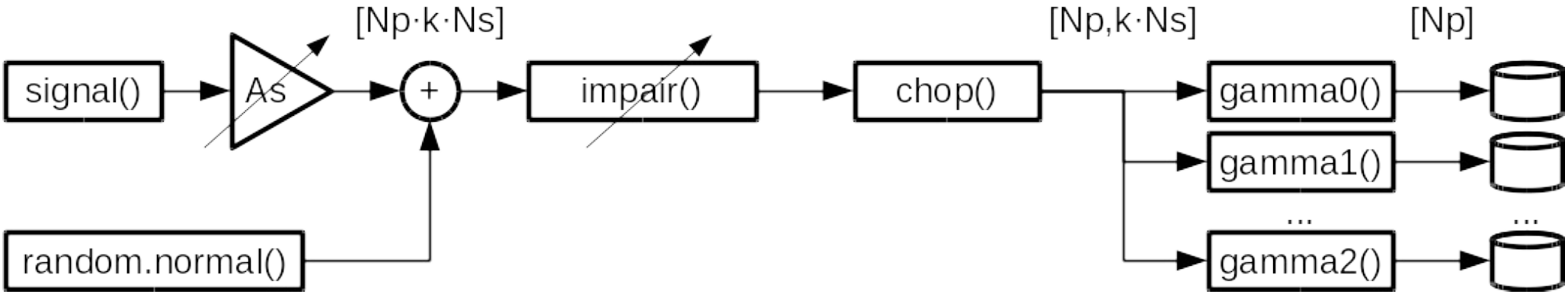


$$f_m = 3.9\text{kHz} \qquad \Delta f = 15.0\text{kHz}$$

C LANTON , C., KENKEL , M., AND TANG , Y. Wireless Microphone Signal Simulation Method. IEEE 802.22-07/0124r0, 2007.

Simulation setup

- Using Python 2.7
  - *numpy* and *scipy* numerical functions
  - *multiprocessing* for creating a process pool
- PRNG – *numpy.random.normal()*
  - Mersenne Twister (uniform distribution)
  - normal PDF obtained through Box-Muller transform



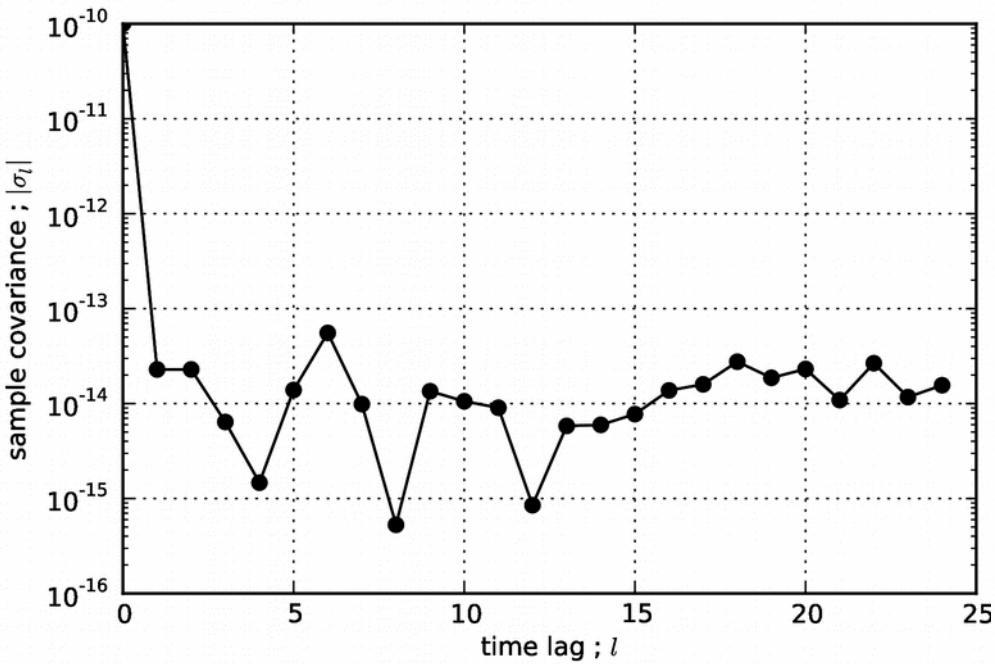
ROY, J.-S. ET AL. *python-numpy-1.6.2/numpy/random/mtrand/randomkit.c*

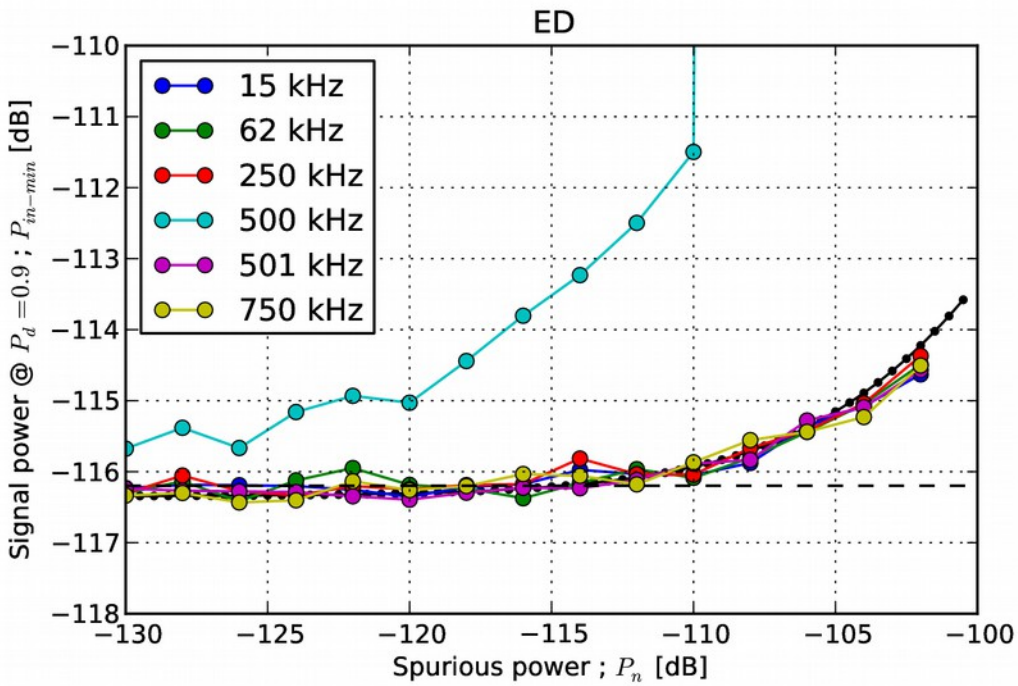
Simulated impairments

- Periodic (cosine) signal at various frequencies
  - $3f_s/8$, $f_s/4 + 1$ kHz, $f_s/4$, $f_s/8$, $f_s/32$, $f_s/128$
  - Model for crosstalk of a clock signal in the circuit
- Simulated ADC oversampling and decimation (DDC)
  - Adjusted N_s, f_s – decimation factors 1, 2 … 8
  - Adjusted σ_w (to keep noise power constant after DDC)
  - *scipy.signal.decimate()* was used (8th order Chebyshev filter)
  - Model for digital down-conversion in digital front-end
- Null

Validation

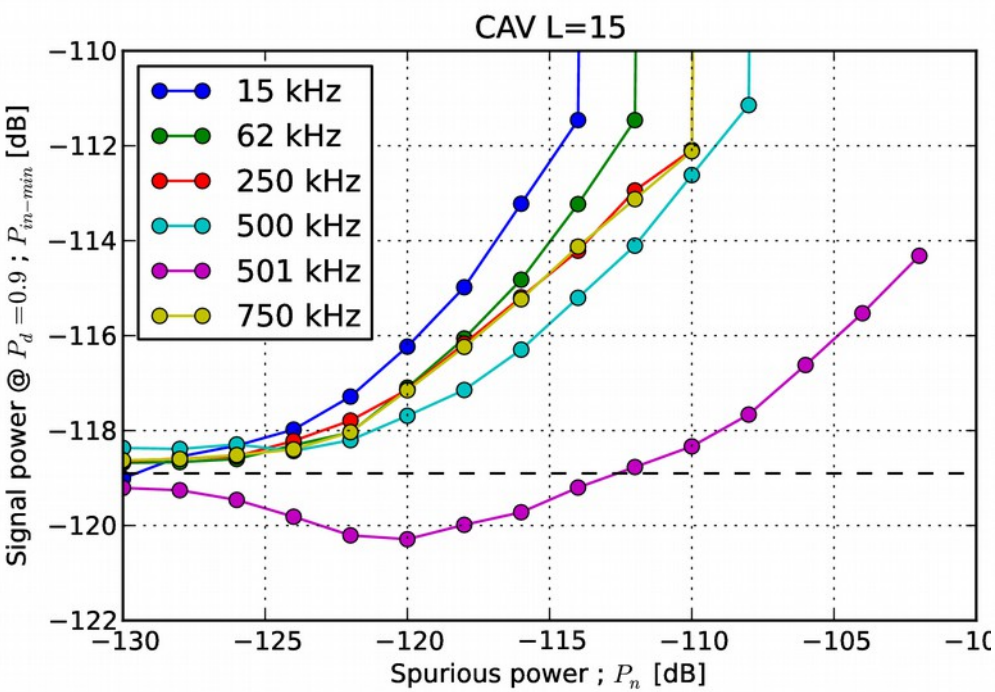
- Check if random samples are uncorrelated
- Check if energy detection results agree with analytical calculation

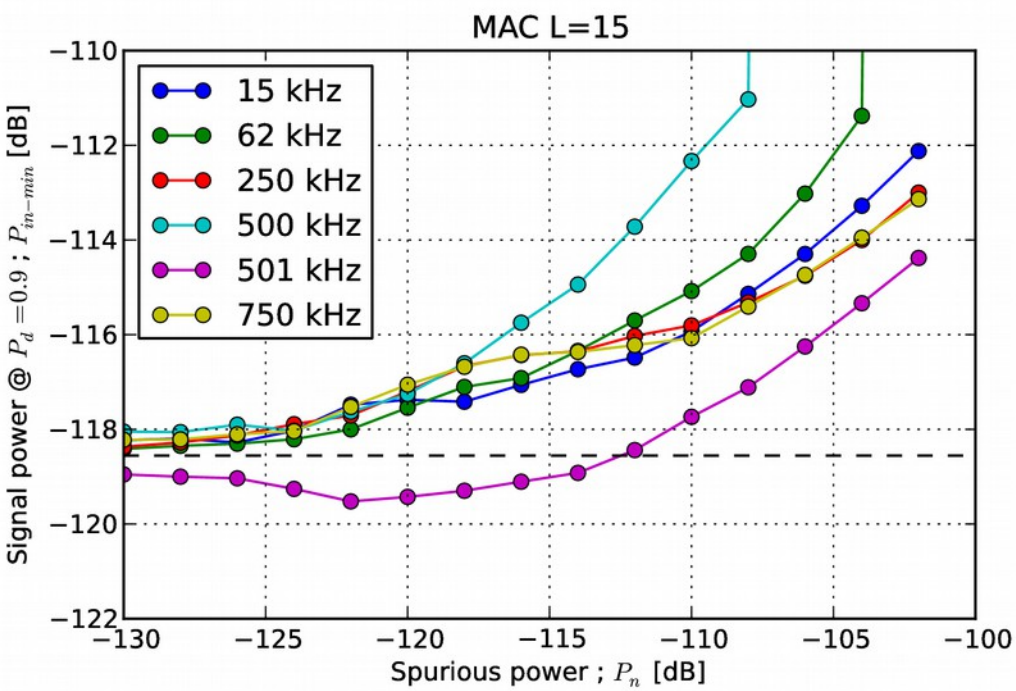




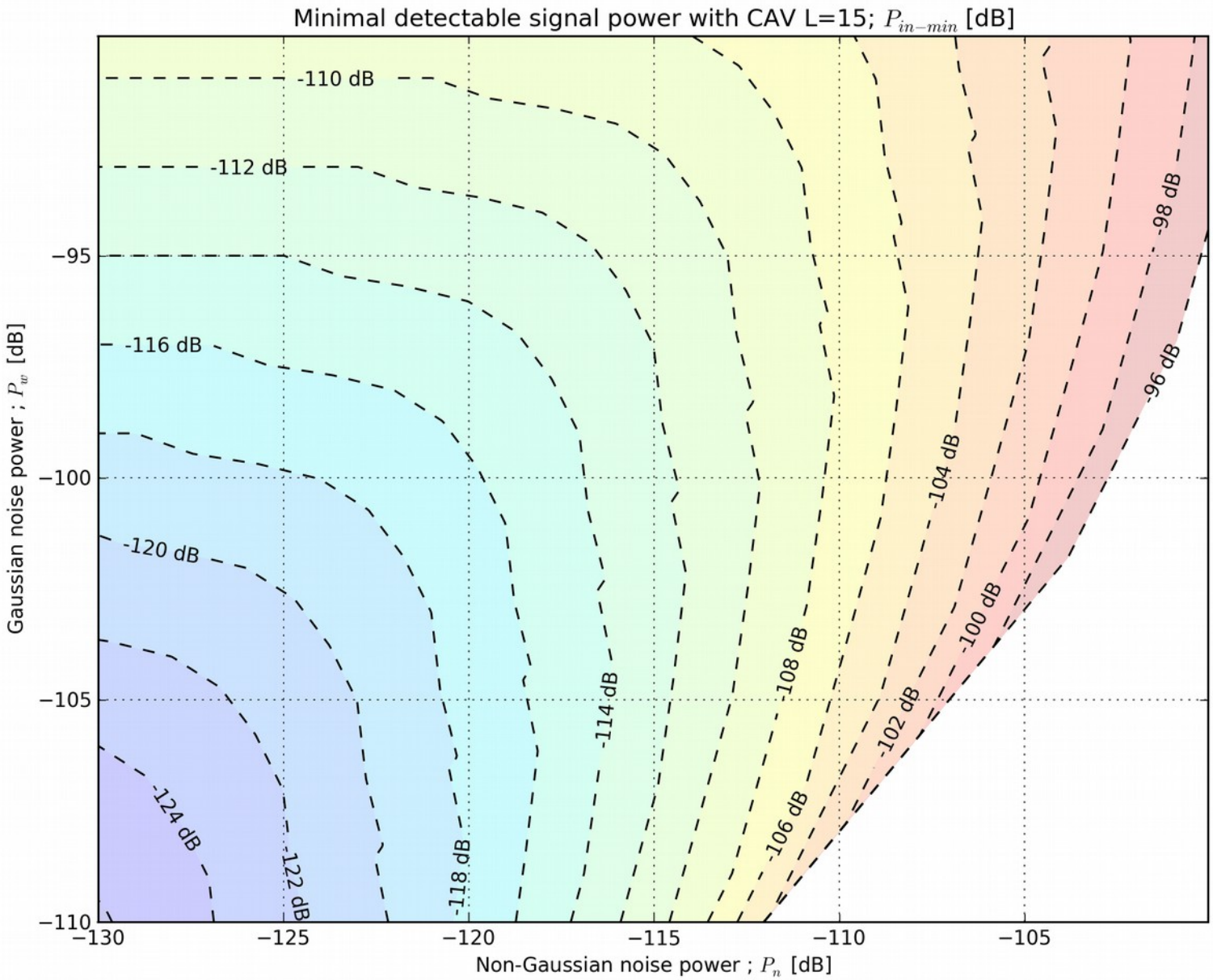
Results – periodic spurious

- Performance degrades much faster than with ED.
- Anomalous performance when signal frequency is at or near spurious frequency.
- MAC detector slightly more resistant than CAV



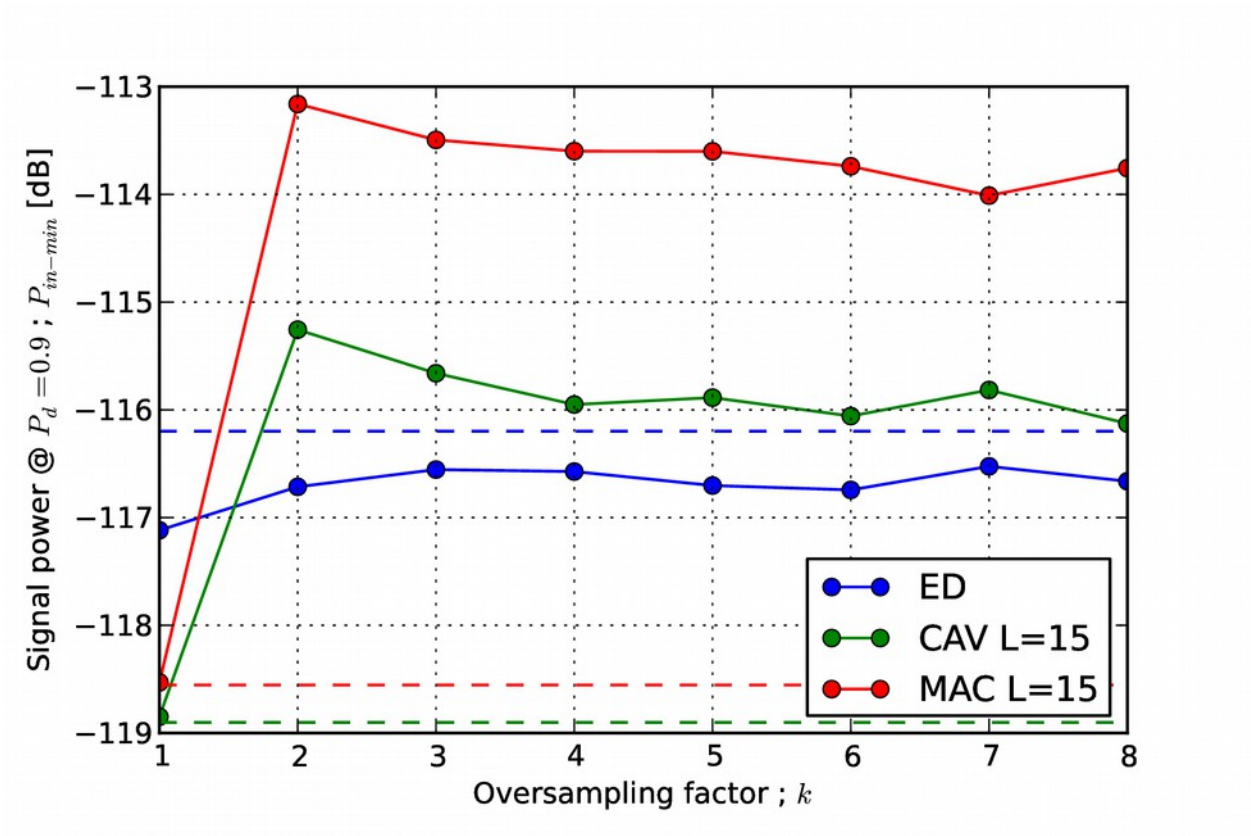


Results – periodic spurious



Results - oversampling

- Causes 3 to 5 dB increase in minimal detectable signal power regardless of *k*.
- CAV detector performs better than MAC.



Conclusions

- Periodic spurious signals significantly affect CBD
  - Only become negligible when below -30 dB compared to Gaussian noise.
  - Performance is decreased even when $f_n << f_s$
  - Create inconsistent detection depending on signal frequency.
- Oversampling also affects CBD, but to a lesser degree
  - Might be corrected using a prewhitening technique.
  - Still most likely a net gain in practice due to reduced total noise power.

Questions?

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